

Khulna University

Science, Engineering and Technology School
Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
Course Code: **0541 12 Math 5101**
Course Title: **Advanced Numerical Analysis**
Total Marks: **60**, Time: **03** Hours, Credit: **03**



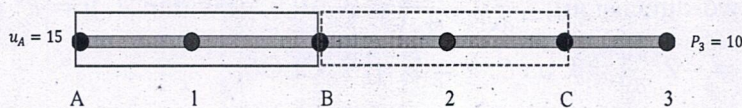
- The figures in the margin indicate full marks. The questions are of equal value.
- Use separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- (a) Explain shooting method for solving boundary value problem. [L2; CLO3] **03**
 - (b) Discuss different types of boundary conditions. [L3; CLO4] **02**
 - (c) Solve the following boundary value problem by Shooting method: $y'' - 2y = 0; y(0) = 1.2, y(1) = 0.9$. Use first initial guess -1.5 and second initial guess -1.0 and $h = 0.25$. [L3; CLO4] **05**
- (a) Solve the initial value problem $y' = t^2(1 + y)$ where $y(1) = 1$ with a step of $h = 0.1$ using Fourth Order Adams-Bashforth Predictor-Corrector method. [L2; CLO2] **05**
 - (b) Using finite difference method, solve the boundary value problem $y'' - 64y + 10 = 0$ with $y(0) = 0, y(1) = 0$. [L3; CLO2] **05**
- (a) Solve by Crank Nicolson method, the equation $u_t = u_{xx}$, given that $u(x, 0) = 100x(1 - x), u(0, t) = 0, u(1, t) = 0$ for one time step taking $h = 0.25$. [L2; CLO3] **05**
 - (b) Solve $y'' - y + x = 0; y(0) = 0 = y(1)$ by applying Rayleigh-Ritz method. [L3; CLO3] **05**
- (a) Define the Finite Volume Method (FVM) and explain its key features. Compare Finite Difference Method (FDM), Finite Element Method (FEM), and Finite Volume Method (FVM) in terms of: Advantages, Disadvantages and Applicability. [L1; CLO4] **05**
 - (b) In the steady, one-dimensional constant-density situation shown in the following figure, the velocity u is calculated for the location A, B and C , while the pressure P is calculated for the location 1, 2 and 3. Also the momentum equations for the grid points B and C are given by the formulas: $u_B = 5 + 2.5(P_1 - P_2)$, $u_C = 5 + 7.5(P_2 - P_3)$ and the boundary conditions are $u_A = 15, P_3 = 10$. Then finding pressure correction equations and calculate P_1, P_2, u_b and u_c by using simple algorithm method (pressure and velocity correction method). [L4; CLO4] **05**

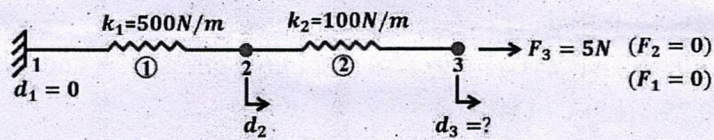


Section-B

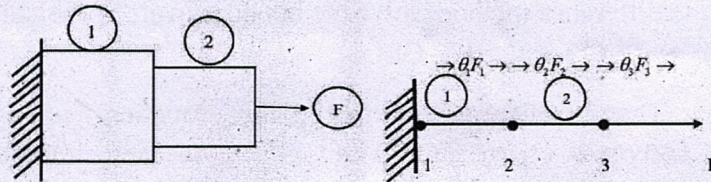
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5. (a) What is Finite Element Method? What is the basic approach of solving problems using Finite Element Method (FEM)? Mention few applications of FEM. [L3; CLO5] 05
- (b) Describe the finite volume method to solve the steady convection-diffusion equation $\frac{d}{dx}(pu\phi) = \frac{d}{dx}(\Gamma \frac{d\phi}{dx})$ for a property ϕ , where the one-dimensional flow field u satisfy the continuity equation $\frac{d}{dx}(pu) = 0$. [L2; CLO5] 05

6. (a) Describe the properties of stiffness matrix. Deduce the reaction force at node-1 from the figure below: [L2; CLO5] 05



- (b) Determine the Reaction forces and stresses (1D FEM for structural Analysis) using the Elimination Approach from the information below: [L4; CLO5] 05



$$E_1 = E_2 = E = 200 \text{ aPa} = 200 \times 10^3 \text{ N/mm}^2, A_1 = 275 \text{ mm}^2, A_2 = 175 \text{ mm}^2, L_1 = 350 \text{ mm}, L_2 = 250 \text{ mm}, F = 700 \text{ kN}$$

7. (a) Solve the following boundary value problem using 1 quadratic element: $\frac{d^2u}{dx^2} = 3, 0 \leq x \leq 2, u(0) = 2, u(2) = \frac{1}{2}$. [L5; CLO5] 05
- (b) Use the standard Galerkin method with one parameter to find an approximate solution to the boundary value problem: $\frac{d^2u}{dx^2} + u = 1, 0 \leq x \leq 1$ with $u(0) = 0, u(1) = 1$. Also compare your results with the exact solution $u(x) = -\cos x + \frac{\cos t}{\sin t} \sin x + 1$. [L5; CLO5] 05
8. (a) Describe the modified Galerkin method to solve the following boundary-value problem (Laplace equation in two-dimension); $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \forall (x, y)$ within $A, u = k^*$ on c^* : $x = 0, y = +2$ and $y = -2, \frac{\partial u}{\partial x} = k$ on $C: x = 2$. [L3; CLO6] 05
- (b) Using a two-parameter trial solution $\tilde{u}(x) = a_1(1 - x^2) + a_2(1 - x^4)$, determine approximate solution to the boundary value problem $\frac{d^2u}{dx^2} + (1 + x^2)u + 1 = 0, -1 \leq x \leq 1, u(-1) = 0, u(1) = 0$. [L4; CLO6] 05

***** THE END *****

*Levels of Bloom's cognitive domain:

[L1=Remember; L2=Understand; L3=Apply; L4=Analyze; L5=Evaluate; L6=Create]

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 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5115**
 Course Title: **Statistical Thermodynamics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- | | | | |
|----|-----|--|----|
| 1. | (a) | Define mean free path. For a large number of molecules, derive the survival relation $dN = N_0 e^{-x/\lambda} dx$, where λ is the mean free path. [L6; CLO1] | 06 |
| | (b) | Distinguish between thermodynamics and statistical thermodynamics? Discuss about statistical equilibrium. [L5; CLO3] | 04 |
| 2. | (a) | Explain classical statistics and quantum statistics. Discuss about microstate and macro state of particles in a thermodynamics system. [L2; CLO2] | 04 |
| | (b) | What is ensemble? Briefly discuss about canonical ensemble, microcanonical ensemble and grand canonical ensemble. [L1; CLO6] | 06 |
| 3. | | Discuss Maxwell-Boltzmann statistics for a system containing a large number of identical and distinguishable particles. Also derive the Maxwell-Boltzmann distribution law for distinguishable particles in terms of temperature. [L2; CLO2] | 10 |
| 4. | (a) | Explain statistical mechanics? Obtain black body radiation of plank from Bose-Einstein statistics. [L2; CLO6] | 05 |
| | (b) | Explain the Maxwell's distribution of molecules velocities. Find the most probable speed, average speed and the root mean square speed using the above distribution. [L2; CLO2] | 05 |

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

- | | | | |
|----|-----|---|----|
| 5. | (a) | What are the properties of fermions? Discuss the quantum statistics for indistinguishable particles of quantum number $\frac{1}{2}$ and $-\frac{1}{2}$. [L3; CLO4] | 05 |
| | (b) | A system consists of 6000 particles distributed in three energy states with equal spacing. The energy of the three states are $E_1 = 0, E_2 = x, E_3 = 2x$. All the three states have the same intrinsic probability g . At a certain instant, there are 3000 particles in the lower level, 2000 particles in the middle level and 1000 particles in the upper level. Compare the relative probabilities with the distribution obtained by the transfer of one particle from middle to lower level and one particle from middle to upper level and the original distribution. [L6; CLO4] | 05 |

6. (a) Using Maxwell-Boltzmann statistics, derive the relations: 05
 (i) $U = KNT^2 \frac{d}{dt}(\log z)$ (ii) $E_{av} = KT^2 \frac{d}{dt}(\log z)$ [L6; CLO5]
- (b) Derive Bose-Einstein distribution function and Fermi-Dirac statistics. 05
 How does Bose-Einstein distribution function differ from Fermi-Dirac statistics. [L6; CLO6].
7. Applying Maxwell-Boltzmann distribution law, show that the internal energy of mono atomic ideal gas depends only on its temperature. [L3; CLO5] 10
8. Derive Bose-Einstein statistics. Also deduce the relation $E(\lambda) = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{\frac{hc}{\lambda T}} - 1} \right]$, where the symbols have their usual meanings. [L6; CLO6] 10

***** THE END *****

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 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5103**
 Course Title: **Signal and Image Processing**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- | | | | |
|----|-----|---|----|
| 1. | (a) | Distinguish between signal and system with example. [L1; CLO1] | 03 |
| | (b) | The input-output relationship of a system is given by $y(n) = 2x(n) + c$, where $x(n)$ is the input, $y(n)$ is the output and c is a constant. Identify whether or not this system is linear. [L3; CLO1] | 04 |
| | (c) | Consider a linear shift-invariant system with an impulse response
$h(n) = \begin{cases} 1; & N_1 \leq n \leq N_2 \\ 0; & \text{otherwise} \end{cases}$ Determine the output of the system when the input sequence is $x(n) = a^n u(n)$ with $ a < 1$. [L5; CLO1] | 03 |
| 2. | (a) | State the sampling theorem. Consider the analog signal $x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$. Determine the Nyquist rate for this signal. [L5; CLO1] | 03 |
| | (b) | Define quantization and quantization error mathematically in the context of analog-to-digital conversion. Is quantization error deterministic or random? [L1; CLO2] | 03 |
| | (c) | State convolution theorem. Perform convolution of the following sequences: $x(n) = [5 \ 2 \ 7 \ -6 \ 10]$, $h(n) = [1 \ -4 \ 3]$. Here the digits in bold represent the signal value at $n = 0$. [L1; CLO1] | 04 |
| 3. | (a) | Define the convolution sum for discrete-time linear time-invariant (LTI) systems and state the properties of the convolution sum: i) commutative, ii) associative, and iii) distributive over addition. [L1; CLO2] | 04 |
| | (b) | Consider a discrete-time LTI system with impulse response $h[n] = 2^n u[-n]$. Using the convolution sum definition, determine whether the system is causal or non-causal. Justify your answer mathematically. [L5; CLO2] | 03 |
| | (c) | Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses $h_1(n) = \left(\frac{1}{2}\right)^n u(n)$, and $h_2(n) = \left(\frac{1}{4}\right)^n u(n)$. [L5; CLO3] | 03 |
| 4. | (a) | Define crosscorrelation and autocorrelation sequences. Compute the autocorrelation of the signal $x(n) = a^n u(n)$, $0 < a < 1$. [L1; CLO3] | 04 |
| | (b) | Solve the second-order difference equation:
$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ for the input $x(n) = 4^n u(n)$. Also derive the total solution $y(n)$, $n > 0$. [L6; CLO3] | 04 |
| | (c) | Determine the pole-zero plot for the signal:
$x(n) = \begin{cases} a^n; & 0 \leq n \leq M-1 \\ 0; & \text{otherwise} \end{cases}, a > 0$ [L5; CLO3] | 02 |

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

- | | | | |
|----|-----|---|----|
| 5. | (a) | Explain the concept of image interpolation with the help of suitable logical or neighborhood-based operations. [L2; CLO4] | 03 |
| | (b) | Discuss the mathematical formulation of affine transformation and explain how operations such as translation, rotation, scaling, and shearing are represented using transformation matrices. [L6; CLO4] | 04 |
| | (c) | Consider three distance metrics: Euclidean, Manhattan, and Chebyshev. For a binary image containing a diagonal line of thickness 1 pixel, compare the results of morphological dilation using a structuring element defined by each distance metric's unit ball. [L4; CLO4] | 03 |
| 6. | (a) | Analyze the second-order derivatives for image sharpening with examples. [L4; CLO4] | 04 |
| | (b) | A medical X-ray image exhibits poor contrast in the region of interest (ROI) while maintaining satisfactory contrast elsewhere. Compare the effectiveness of global histogram equalization and adaptive histogram equalization for enhancing the ROI. [L4; CLO4] | 03 |
| | (c) | Write the mathematical formulas for the band-pass filter and the band-reject filter kernels, and explain how each filter affects the quality and features of an image. [L1; CLO4] | 03 |
| 7. | (a) | A two-dimensional discrete image is represented in the spatial domain by $f(x, y)$. Derive the mathematical expressions for the 2-D Discrete Fourier Transform (DFT) and the Inverse Discrete Fourier Transform (IDFT) used in frequency-domain image processing. [L3; CLO4] | 02 |
| | (b) | Discuss the fundamental properties of the 2-D DFT, and explain their significance in digital image analysis. [L5; CLO5] | 05 |
| | (c) | Compare and explain the probability density functions of Gaussian noise, Salt-and-Pepper noise, and Erlang noise commonly encountered in digital image processing. [L4; CLO5] | 03 |
| 8. | (a) | Construct the red, green, blue (RGB), and hue, saturation, intensity (HIS) color models for color image processing. [L3; CLO5] | 04 |
| | (b) | Discuss the color edge detection from the RGB image. [L6; CLO5] | 03 |
| | (c) | Explain the basic steps of the image segmentation process. [L5; CLO5] | 03 |

***** THE END *****

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 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5105**
 Course Title: **Fluid Dynamics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Derive the Navier-Stokes equation and continuity equation for compressible viscous fluid in vector form: $\frac{\partial q}{\partial t} + q(q \cdot \nabla) = F - \frac{1}{\rho} \nabla P + \nu \nabla^2 q + \frac{1}{3} \nu \nabla(\nabla \cdot q)$. [L5; CLO1] **05**
 (b) Define Couette flow. Show that for Couette flow, the solution of Navier Stokes equation is $u = \frac{y}{h} U - \frac{h^2}{2\mu} \frac{dp}{dx} \frac{y}{h} \left(1 - \frac{y}{h}\right)$, where the symbols have their usual meaning. Also find the shearing stresses. [L2; CLO2] **05**
2. (a) What is a parallel flow? Show that for parallel flow through a straight channel, resulting velocity profile is parabolic. [L2; CLO3] **05**
 (b) Find the velocity distribution of laminar steady flow between two co-axial circular cylinders. [L1; CLO2] **05**
3. (a) Define unsteady flow. Obtain velocity distribution as well as drag coefficients for a flow past a suddenly accelerated flat plate. [L2; CLO4] **05**
 (b) An incompressible viscous fluid flows steadily parallel to the axis in an annular space between two coaxial cylinders of radii a and b ($b > a$). Show that the rate of discharge is $-\frac{\pi P}{8\mu} \left[(b^4 - a^4) - \frac{(b^2 - a^2)^2}{\log \frac{b}{a}} \right]$ **05**
 where, P is the pressure gradient. Also, find the average velocity. [L2; CLO1]
4. Show that the stream function ψ for the slow motion of viscous fluid streaming past a sphere of radius ' a ' is given by $\psi = \frac{U}{2} \left(r^2 - \frac{3}{2} ar + \frac{1}{2} \frac{a^3}{r} \right) \sin^2 \theta$. [L2; CLO3] **10**

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. Show that with a suitable substitution, the boundary layer equation over a flat plate reduces to the Blasius equation in the form: $f(\eta)f''(\eta) + 2f'''(\eta) = 0$ with boundary conditions $f(0) = f'(0) = 0$ and $f'(\infty) = 1$, **10**

$$\text{where } \eta = \sqrt{\frac{U_\infty}{\nu x}} y, \psi = \sqrt{\nu U_\infty x} f(\eta)$$

Also, show that the skin friction for a plate wetted on both sides is $4f''(0)b\sqrt{\mu\rho lU^3}$, where b is the width and l is the length of the plate. [L2; CLO4]

6. Obtain the integral equation for an unsteady boundary layer flow which is related to the displacement thickness and momentum thickness. [L4; CLO5] **10**

7. Find the Boundary Layer Equation for steady 2D incompressible fluid flow in a convergent/divergent channel, and hence show that **10**

$$f' = \frac{U}{U_\infty} = 3 \tanh^2 \left\{ \frac{\eta}{\sqrt{2}} + \beta \right\} - 2. \quad [\text{L2; CLO6}]$$

8. Explain forced convection and free convection. Describe the parallel forced flow past a flat plate at zero incidence and hence obtain the solution for the 'cooling problem' with a prescribed temperature difference between the wall and external stream. [L6; CLO7] **10**

***** THE END *****

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 Year: **MSc First, Term: I Examination-2026; Session: 2025-2026**
 Course Code: **0541 12 Math 5111**
 Course Title: **Optimal Control and Applications**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- | | | | |
|----|-----|--|----|
| 1. | (a) | Briefly discuss the optimization problems and optimal control problems with examples. Distinguish the basic difference between the two types of problems. [L1; CLO1] | 04 |
| | (b) | Define process, optimal process, feasible solution and optimal solution of optimal control problem. Show graphically the main idea of optimal solution. [L1; CLO1] | 03 |
| | (c) | Describe the inter-relations among different types of optimal control problems. [L2; CLO2] | 03 |
| 2. | (a) | Distinguish between function and functional with examples. Discuss briefly the calculus of variations technique. Derive the necessary condition for a functional of 3 rd order derivatives to be extremal. [L5; CLO3] | 06 |
| | (b) | State the fundamental lemma of calculus of variations. Prove the lemma by applying the 'way of contradiction (WOC)' technique. [L4; CLO2, CLO3] | 04 |
| 3. | (a) | Define piecewise differentiable function, Lipschitz continuous function, bounded function and convex function for two variables. [L5; CLO3] | 03 |
| | (b) | Distinguish between algebra and σ -algebra, measure and measurable set. [L3; , CLO3, CLO4] | 03 |
| | (c) | State the integral form of Gronwall's lemma and prove the lemma. [L2; CLO2] | 04 |
| 4. | (a) | Derive the necessary conditions to solve the following optimal control problem:
$\min_u \int_{t_0}^{t_1} f(t, x(t), u(t)) dt$ subject to $x'(t) = g(t, x(t), u(t))$, $x(t_0) = x_0$ and $x(t_1)$ free. [L3; CLO3] | 06 |

- (b) An optimal control problem is as follows:

04

$$J(u) = \int_{t_0}^{t_1} f(t, x(t), u(t)) dt$$

subject to $x'(t) = g(t, x(t), u(t))$, $x(t_0) = x_0$.

Consider, $f(t, x, u)$ and $g(t, x, u)$ are both continuously differentiable functions in their three arguments and concave in x and u . Suppose u^* is a control, with associated state x^* , and λ a piecewise differentiable function, such that u^* , x^* , and λ together satisfy on $t_0 \leq t \leq t_1$: $f_u + \lambda g_u = 0$, $\lambda' = -(f_x + \lambda g_x)$, $\lambda(t_1) = 0$, $\lambda(t) \geq 0$

Then for all controls u , we have to prove that $J(u^*) \geq J(u)$. [L3; CLO3]

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) Discuss the main differences between necessary conditions of optimality and Pontryagin's maximum principle. Derive the set of necessary conditions for a basic optimal control problem. [L2; CLO2] 06
 (b) Prove that the Hamiltonian is a Lipschitz continuous function of time t on the optimal path. [L4; CLO1] 04

6. (a) Characterize the following optimal control problem for an optimal solution: 04

$$\min_u \int_0^2 (x(t) - t^2)^2 dt$$
 subject to $x'(t) = u(t), x(0) = 1, 0 \leq u(t) \leq 4$. [L3; CLO4]
 (b) Determine the optimal solution of the following optimal control problem: 06

$$\min_{u_1, u_2} \int_0^1 (x(t) - \frac{1}{8}u_1^2(t) - \frac{1}{2}u_2^2(t))dt$$
 subject to $x'(t) = u_1(t) + u_2(t), x(0) = 0, 1 \leq u(t) \leq 2$. [L5; CLO3]

7. (a) Consider an optimal control problem dealing with a one-sided control constraint 05

$$\max x(4) - \int_0^4 u^2(t) dt \quad \text{subject to} \quad \dot{x}(t) = x(t) + u(t), x(0) = 0, u(t) \leq 5.$$
 Write down the Hamiltonian and solve for the optimal solution. [L5; CLO4, CLO5]
 (b) Let $x(t)$ represents the number of cancer cells at time t (with exponential growth factor Q), and $u(t)$ the drug concentration. Suppose you want to minimize the number of cancer cells at the end of the treatment period and at the same time the harmful effects of the drug on the body. Formulate the optimal control problem and solve it. [L4; CLO3] 05

8. Consider a micro-parasitic infectious disease in human population, where permanent immunity to the disease can be achieved through natural recovery. Suppose you wish to minimize the number of infectious persons by a vaccination program for a certain fixed time period and at the same time to minimize the overall cost of vaccination. Formulate a FOUR compartmental SEIRN model for the vaccination schedule of this epidemic disease and solve for the optimal control u^* in closed form. [L5, L6; CLO4, CLO5] 10

***** THE END *****

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 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0111 12 IER 5159**
 Course Title: **Research Methodology and Ethical Issues**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

		Marks
1.	(a) Explain the difference between basic research and applied research. Provide one example of each from mathematical field. [L2; CLO2, L3; CLO4]	04
	(b) Discuss the role of deductive and inductive reasoning in research. Which one is more dominant in mathematics and why? [L3, L4; CLO4]	02
	(c) What is a research gap? And why is it important to know in any research? [L2; CLO1]	02
	(d) How to create a research problem based on the literature review? [L1; CLO3]	02
2.	(a) Compare quantitative and qualitative research approaches. Which is more relevant to mathematics and in what contexts? [L4; CLO2, CLO6]	04
	(b) How do you differentiate between theoretical and empirical research? [L2; CLO3]	02
	(c) Describe the key steps of writing a research proposal. [L6; CLO3]	04
3.	(a) Differentiate between a hypothesis and a conjecture. Why are conjectures more common in mathematics? [L2; CLO7]	04
	(b) Miss X is conducting research on "A Mathematical Model for the Spread of Infectious Diseases with Vaccination". Can you write objectives and conjectures? [L6; CLO10]	06
4.	(a) What is scale and measurement in Research Methodology? [L1; CLO4]	02
	(b) Differentiate between Citation, Reference and Bibliography with example. [L5; CLO9]	03
	(c) Differentiate between 'proof by contradiction' and 'proof by contrapositive'. [L5; CLO9]	05

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5.	(a) What is 'Exposition' in mathematical writing? [L2; CLO4]	02
	(b) Could you list different types of expository writing and explain any two of them. [L2; CLO4]	03
	(c) Compare and contrast between exposition and research paper. [L4; CLO4]	02
	(d) Differentiate between Citation, Reference and Bibliography with example. [L5; CLO9]	03

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|----|-----|--|----|
| 6. | (a) | What is plagiarism? [L2; CLO8] | 01 |
| | (b) | Differentiate between direct plagiarism and paraphrasing plagiarism with examples. [L3; CLO8] | 04 |
| | (c) | A student copies a proof from a textbook and changes only a few words before submission. | 05 |
| | | (i) What type of plagiarism is this? | |
| | | (ii) Why is it unethical? | |
| | | (iii) How could the student avoid plagiarism? [L3; CLO8] | |
| 7. | (a) | What is meant by authorship in research? [L2; CLO8] | 02 |
| | (b) | Explain the responsibilities of an author in mathematical research. [L4; CLO8] | 02 |
| | (c) | A professor's name is added to a paper even though they made no contribution to the research. | 06 |
| | | (i) What type of unethical authorship practice is this? | |
| | | (ii) Why is it problematic? | |
| | | (iii) What should be done instead? [L5; CLO8] | |
| 8. | (a) | What is the dissemination of research? Why is the dissemination of research important? [L5; CLO8] | 05 |
| | (b) | What are the indicators of disseminating research? How can a graphical abstract help disseminate your research? [L5; CLO8] | 05 |

***** THE END *****

*Levels of Bloom's cognitive domain:

[L1=Remember; L2=Understand; L3=Apply; L4=Analyze; L5=Evaluate; L6=Create]

Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I Examination-2026**; Session: **2025-2026**
 Course Code: **0541 12 Math 5107**
 Course Title: **Industrial Mathematics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the margin indicate full marks. The questions are of equal value.
- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Define the probability density function, probability mass function and expected value and how they measure? [L1; CLO1] **04**
 (b) Define Poisson distribution. What is the relation between Poisson and Binomial distributions? [L2; CLO1] **02**
 (c) 20% chance of suffering of occupational diseases in an industry. What is the probability that out of 6 workers i) 3 ii) 4 iii) 4 or more workers will contract the diseases? [L1; CLO1] **04**
2. (a) Define Z- transformation of a sequence. Find the Z- transformation of **05**
 i. discrete unit set $U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$
 ii. $U(x) = \{15, 10, 14, 7, 4, 1, -1, 0, 3\}$ [L2; CLO2]
 (b) Define casual signals. Solve the recursion: $6u_k = 5u_{k-1} - u_{k-2}$ when $u_0 = 1$ and $u_1 = -1$. [L5; CLO2] **05**
3. (a) Define Discrete Fourier Transform (DFT) and IDFT. [L1; CLO2] **02**
 (b) If $F[k] = \{2, 1 - i, 0, 1 + i\}$ is the frequency domain signal, find its time domain signal $f[n]$. [L2; CLO2] **03**
 (c) Using convolution theorem, determine circular convolution of $f[n] = \{1, 1, 0, 0\}$ and $g[n] = -1^n, n = 0, 1, 2, 3$. [L5; CLO3] **05**
4. (a) Define marginal cost and marginal revenue. [L2; CLO3] **02**
 (b) Show that the transformation of convolution is co-ordinate wise product of the two transformations. [L3; CLO3] **03**
 (c) For what condition $\psi(x, a) = a_1\phi_1 + a_2\phi_2 + \dots + a_n\phi_n$ will be the best fit for the data. [L5; CLO4] **05**

x	x_1	x_2	$\dots$	x_n
y	y_1	y_2	$\dots$	y_n

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) What is depreciation and annuity due? Let an investment machine cost is 5,00,000Tk and its effective life is estimated 15 years. If the scrap realizes 5,000Tk only. What amount should be retained out of profits at the end of each year to accumulate at 5% compound interest per annum? [L5; CLO4] **05**
- (b) A company's production cost is $C(t)$ over time in year is governed by the DE, $\frac{dC}{dt} = 15,000 - 0.05C$, with an initial cost $C(0) = \text{Tk.}50,000$. The company's revenue is given by $R(t) = 300t + 2500$. Determine the cost function and the profit at $t = 4$. [L5; CLO4] **05**
6. (a) An attack submarine is cruising at 40 knots/hour at a depth of 1000 feet when suddenly the reactor scrams. After 1 minute way has dropped to 30 knots/hour. Construct a mathematical model of the submarine. How long does the crew have to make repairs before forward motion falls below steerageway of 2 knots/hour? [L3; CLO5] **05**
- (b) List the Big Six Partial Differential Equations (PDEs) and describe their significance in mathematical modeling. [L2; CLO5] **05**
7. (a) Explain the plant cascade model and illustrate its application in control systems. [L2; CLO5] **05**
- (b) Why numerical methods are important for solving PDEs and what types of problem are solved by this method? Using numerical method solve: $u_t = u_{xx}$, $0 < x < 1$, subject to $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 1$. [L5; CLO6] **05**
8. (a) Two cascade plants, 1st one is lowpass filter given by $\dot{y} + y = u$ and 2nd one is high-pass filter given by $\dot{z} + z = y$. Solve the system for $y(t)$ using frequency domain method. [L5; CLO6] **05**
- (b) What is Galerkin's requirement? By using Galerkin's method solve the eigen value problem: $y'' = \lambda y$, $0 < x < 1$ subject to $y(0) = 0 = y(1)$. [L5; CLO6] **05**

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: Mathematics, Program: M.Sc. in Applied Mathematics
 Year: MSc First, Term: I Examination-2026; Session: 2025-2026
 Course Code: 0541 12 Math 5109
 Course Title: Operations Research
 Total Marks: 60, Time: 03 Hours, Credit: 03



- The figures in the margin indicate full marks. The questions are of equal value.
- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Briefly discuss about the different phases of operations research (OR). Also describe its limitations. [L1; CLO1] 05
 (b) Solve the following linear goal programming problem: 05
 Minimize $Z = \{o_3 + o_4, o_1, u_2, u_3 + 8/5u_4\}$,
 subject to $g_1: x_1 + x_2 + u_1 - o_1 = 20$
 $g_2: x_1 + x_2 + u_2 - o_2 = 50$
 $g_3: x_1 + u_3 - o_3 = 15$
 $g_4: x_2 + u_4 - o_4 = 8$
 $x_i, u_i, o_i \geq 0$
 Goals are written in order of priority. [L2; CLO1]
2. (a) Explain the term 'Goal Deviation' in goal programming. [L2; CLO2] 03
 (b) Solve the following goal programming problem: A company manufactures two products, radios and televisions, which must be processed through assembly and finishing departments. Assembly has 90 hours available finishing can handle up to 72 hours of work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each television requires 3 hours in assembly and 6 hours in finishing. If profit is Tk. 120 per radio and Tk. 90 per television, determine the best combination of radios and televisions to realize a maximum profit of Tk. 2100. [L5; CLO2] 07
3. Solve the following mixed integer programming problem: 10
 Maximize $z = 6x_1 + 9x_2$
 Subject to $-x_1 + 3x_2 \leq 6$
 $7x_1 + x_2 \leq 35$
 $x_1, x_2 \geq 0$ and x_1 an integer. [L1; CLO4]
4. (a) Briefly explain about dynamic programming problem. [L4; CLO4] 03
 (b) Evaluate optimal solution of the following problem with help of dynamic programming process: 07
 $Min Z = y_1^2 + y_2^2 + y_3^2$
 Subject to: $y_1 + y_2 + y_3 = 10; y_1, y_2, y_3 \geq 0$. [L6; CLO4]

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) What do you mean by decision theory? Discuss taking decision making under certainty, under risk, and under uncertainty. [L1; CLO1] 04
- (b) The following matrix gives the payoff of different alternatives a_1, a_2 and a_3 against states of nature (events) S_1, S_2, S_3 and S_4 . Discuss the decision taken under the following approach (i) Pessimistic (ii) Optimistic [L4; CLO2] 06

	S_1	S_2	S_3	S_4
a_1	Tk. 4,000	Tk. -100	Tk. 6,000	Tk. 18,000
a_2	Tk. 20,000	Tk. 5,000	Tk. 400	Tk. 0
a_3	Tk. 20,000	Tk. 15,000	Tk. -2,000	Tk. 1,000

6. (a) Solve the following 2×2 game by arithmetic method: [L3; CLO4] 05

		Player B	
		B_1	B_2
Player A	A_1	2	-1
	A_2	-1	0

- (b) Solve the following game by graphical method: [L5; CLO5] 05

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	2	1	0	-2
	A_2	1	0	3	2

7. (a) Use dominance rule to reduce the following game to 2×2 game and hence find the optimum strategies and the value of the game for player A and B. [L3; CLO6] 05

	B_1	B_2	B_3
A_1	3	-2	4
A_2	-1	4	2
A_3	2	2	6

- (b) Two players A and B play the following game. A has a bag containing three coins of 1 Tk, 2 Tk, and 5 Tk. A Takes one coin from the bag and before it is exposed, B guesses what it is? If he, B is right he takes the coin if he is wrong, he gives to A a coin of the same taka. Solve the game. [L4; CLO6] 05

8. Solve the following non-linear programming problem (NLPP) by using Kuhn-Tucker conditions 10

$$\begin{aligned} \text{Optimize } Z &= 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2) \\ \text{Subject to: } x_1 + x_2 &\leq 1; 2x_1 + 3x_2 \leq 6; x_1, x_2 \geq 0. \end{aligned} \quad [L6; CLO5]$$

***** THE END *****

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Khulna University

Science, Engineering and Technology School

Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**

Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**

Course Code: **0541 12 Math 5101**

Course Title: **Advanced Numerical Analysis**

Total Marks: **60**, Time: **03** Hours, Credit: **03**



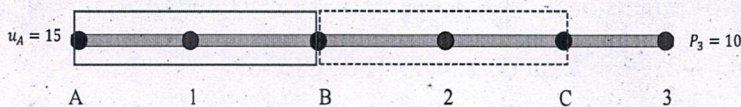
- The figures in the margin indicate full marks. The questions are of equal value.
- Use separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- (a) Explain shooting method for solving boundary value problem. [L2; CLO3] **03**
 - (b) Discuss different types of boundary conditions. [L3; CLO4] **02**
 - (c) Solve the following boundary value problem by Shooting method: $y'' - 2y = 0; y(0) = 1.2, y(1) = 0.9$. Use first initial guess -1.5 and second initial guess -1.0 and $h = 0.25$. [L3; CLO4] **05**
- (a) Solve the initial value problem $y' = t^2(1 + y)$ where $y(1) = 1$ with a step of $h = 0.1$ using Fourth Order Adams-Bashforth Predictor-Corrector method. [L2; CLO2] **05**
 - (b) Using finite difference method, solve the boundary value problem $y'' - 64y + 10 = 0$ with $y(0) = 0, y(1) = 0$. [L3; CLO2] **05**
- (a) Solve by Crank Nicolson method, the equation $u_t = u_{xx}$, given that $u(x, 0) = 100x(1 - x), u(0, t) = 0, u(1, t) = 0$ for one time step taking $h = 0.25$. [L2; CLO3] **05**
 - (b) Solve $y'' - y + x = 0; y(0) = 0 = y(1)$ by applying Rayleigh-Ritz method. [L3; CLO3] **05**
- (a) Define the Finite Volume Method (FVM) and explain its key features. Compare Finite Difference Method (FDM), Finite Element Method (FEM), and Finite Volume Method (FVM) in terms of: Advantages, Disadvantages and Applicability. [L1; CLO4] **05**
 - (b) In the steady, one-dimensional constant-density situation shown in the following figure, the velocity u is calculated for the location A, B and C , while the pressure P is calculated for the location 1, 2 and 3. Also the momentum equations for the grid points B and C are given by the formulas: $u_B = 5 + 2.5(P_1 - P_2)$, $u_C = 5 + 7.5(P_2 - P_3)$ and the boundary conditions are $u_A = 15, P_3 = 10$. Then finding pressure correction equations and calculate P_1, P_2, u_b and u_c by using simple algorithm method (pressure and velocity correction method). [L4; CLO4] **05**

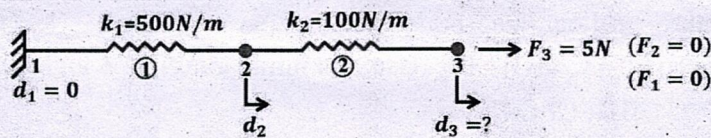


Section-B

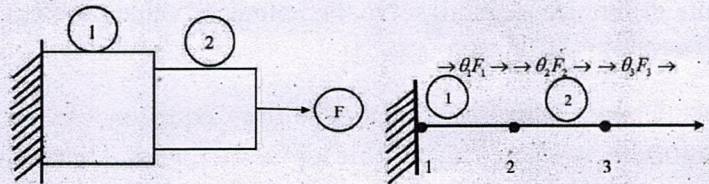
There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) What is Finite Element Method? What is the basic approach of solving problems using Finite Element Method (FEM)? Mention few applications of FEM. [L3; CLO5] 05
- (b) Describe the finite volume method to solve the steady convection-diffusion equation $\frac{d}{dx}(pu\phi) = \frac{d}{dx}(\Gamma \frac{d\phi}{dx})$ for a property ϕ , where the one-dimensional flow field u satisfy the continuity equation $\frac{d}{dx}(pu) = 0$. [L2; CLO5] 05

6. (a) Describe the properties of stiffness matrix. Deduce the reaction force at node-1 from the figure below: [L2; CLO5] 05



- (b) Determine the Reaction forces and stresses (1D FEM for structural Analysis) using the Elimination Approach from the information below: [L4; CLO5] 05



$$E_1 = E_2 = E = 200 \text{ aPa} = 200 \times 10^3 \text{ N/mm}^2, A_1 = 275 \text{ mm}^2, A_2 = 175 \text{ mm}^2, L_1 = 350 \text{ mm}, L_2 = 250 \text{ mm}, F = 700 \text{ kN}$$

7. (a) Solve the following boundary value problem using 1 quadratic element: $\frac{d^2u}{dx^2} = 3, 0 \leq x \leq 2, u(0) = 2, u(2) = \frac{1}{2}$. [L5; CLO5] 05
- (b) Use the standard Galerkin method with one parameter to find an approximate solution to the boundary value problem: $\frac{d^2u}{dx^2} + u = 1, 0 \leq x \leq 1$ with $u(0) = 0, u(1) = 1$. Also compare your results with the exact solution $u(x) = -\cos x + \frac{\cos t}{\sin t} \sin x + 1$. [L5; CLO5] 05
8. (a) Describe the modified Galerkin method to solve the following boundary-value problem (Laplace equation in two-dimension); $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \forall (x, y)$ within $A, u = k^*$ on c^* : $x = 0, y = +2$ and $y = -2, \frac{\partial u}{\partial x} = k$ on $C: x = 2$. [L3; CLO6] 05
- (b) Using a two-parameter trial solution $\tilde{u}(x) = a_1(1 - x^2) + a_2(1 - x^4)$, determine approximate solution to the boundary value problem $\frac{d^2u}{dx^2} + (1 + x^2)u + 1 = 0, -1 \leq x \leq 1, u(-1) = 0, u(1) = 0$. [L4; CLO6] 05

***** THE END *****

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[L1=Remember; L2=Understand; L3=Apply; L4=Analyze; L5=Evaluate; L6=Create]

Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5115**
 Course Title: **Statistical Thermodynamics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the right margin indicate full marks. The questions are of equal value.
- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

- | | | | |
|----|-----|--|----|
| 1. | (a) | Define mean free path. For a large number of molecules, derive the survival relation $dN = N_0 e^{-x/\lambda} dx$, where λ is the mean free path. [L6; CLO1] | 06 |
| | (b) | Distinguish between thermodynamics and statistical thermodynamics? Discuss about statistical equilibrium. [L5; CLO3] | 04 |
| 2. | (a) | Explain classical statistics and quantum statistics. Discuss about microstate and macro state of particles in a thermodynamics system. [L2; CLO2] | 04 |
| | (b) | What is ensemble? Briefly discuss about canonical ensemble, microcanonical ensemble and grand canonical ensemble. [L1; CLO6] | 06 |
| 3. | | Discuss Maxwell-Boltzmann statistics for a system containing a large number of identical and distinguishable particles. Also derive the Maxwell-Boltzmann distribution law for distinguishable particles in terms of temperature. [L2; CLO2] | 10 |
| 4. | (a) | Explain statistical mechanics? Obtain black body radiation of plank from Bose-Einstein statistics. [L2; CLO6] | 05 |
| | (b) | Explain the Maxwell's distribution of molecules velocities. Find the most probable speed, average speed and the root mean square speed using the above distribution. [L2; CLO2] | 05 |

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

- | | | | |
|----|-----|---|----|
| 5. | (a) | What are the properties of fermions? Discuss the quantum statistics for indistinguishable particles of quantum number $\frac{1}{2}$ and $-\frac{1}{2}$. [L3; CLO4] | 05 |
| | (b) | A system consists of 6000 particles distributed in three energy states with equal spacing. The energy of the three states are $E_1 = 0, E_2 = x, E_3 = 2x$. All the three states have the same intrinsic probability g . At a certain instant, there are 3000 particles in the lower level, 2000 particles in the middle level and 1000 particles in the upper level. Compare the relative probabilities with the distribution obtained by the transfer of one particle from middle to lower level and one particle from middle to upper level and the original distribution. [L6; CLO4] | 05 |

6. (a) Using Maxwell-Boltzmann statistics, derive the relations: 05
 (i) $U = KNT^2 \frac{d}{dt}(\log z)$ (ii) $E_{av} = KT^2 \frac{d}{dt}(\log z)$ [L6; CLO5]
- (b) Derive Bose-Einstein distribution function and Fermi-Dirac statistics. 05
 How does Bose-Einstein distribution function differ from Fermi-Dirac statistics. [L6; CLO6].
7. Applying Maxwell-Boltzmann distribution law, show that the internal energy of mono atomic ideal gas depends only on its temperature. [L3; CLO5] 10
8. Derive Bose-Einstein statistics. Also deduce the relation $E(\lambda) = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{\frac{hc}{\lambda T}} - 1} \right]$, where the symbols have their usual meanings. [L6; CLO6] 10

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5103**
 Course Title: **Signal and Image Processing**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the margin indicate full marks. The questions are of equal value.
- Use separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

	Marks
1. (a) Distinguish between signal and system with example. [L1; CLO1]	03
(b) The input-output relationship of a system is given by $y(n) = 2x(n) + c$, where $x(n)$ is the input, $y(n)$ is the output and c is a constant. Identify whether or not this system is linear. [L3; CLO1]	04
(c) Consider a linear shift-invariant system with an impulse response $h(n) = \begin{cases} 1; & N_1 \leq n \leq N_2 \\ 0; & \text{otherwise} \end{cases}$ Determine the output of the system when the input sequence is $x(n) = a^n u(n)$ with $ a < 1$. [L5; CLO1]	03
2. (a) State the sampling theorem. Consider the analog signal $x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$. Determine the Nyquist rate for this signal. [L5; CLO1]	03
(b) Define quantization and quantization error mathematically in the context of analog-to-digital conversion. Is quantization error deterministic or random? [L1; CLO2]	03
(c) State convolution theorem. Perform convolution of the following sequences: $x(n) = [5 \ 2 \ 7 \ -6 \ 10]$, $h(n) = [1 \ -4 \ 3]$. Here the digits in bold represent the signal value at $n = 0$. [L1; CLO1]	04
3. (a) Define the convolution sum for discrete-time linear time-invariant (LTI) systems and state the properties of the convolution sum: i) commutative, ii) associative, and iii) distributive over addition. [L1; CLO2]	04
(b) Consider a discrete-time LTI system with impulse response $h[n] = 2^n u[-n]$. Using the convolution sum definition, determine whether the system is causal or non-causal. Justify your answer mathematically. [L5; CLO2]	03
(c) Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses $h_1(n) = \left(\frac{1}{2}\right)^n u(n)$, and $h_2(n) = \left(\frac{1}{4}\right)^n u(n)$. [L5; CLO3]	03
4. (a) Define crosscorrelation and autocorrelation sequences. Compute the autocorrelation of the signal $x(n) = a^n u(n)$, $0 < a < 1$. [L1; CLO3]	04
(b) Solve the second-order difference equation: $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ for the input $x(n) = 4^n u(n)$. Also derive the total solution $y(n)$, $n > 0$. [L6; CLO3]	04
(c) Determine the pole-zero plot for the signal: $x(n) = \begin{cases} a^n; & 0 \leq n \leq M-1 \\ 0; & \text{otherwise} \end{cases}, a > 0$ [L5; CLO3]	02

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

- | | | | |
|----|-----|---|----|
| 5. | (a) | Explain the concept of image interpolation with the help of suitable logical or neighborhood-based operations. [L2; CLO4] | 03 |
| | (b) | Discuss the mathematical formulation of affine transformation and explain how operations such as translation, rotation, scaling, and shearing are represented using transformation matrices. [L6; CLO4] | 04 |
| | (c) | Consider three distance metrics: Euclidean, Manhattan, and Chebyshev. For a binary image containing a diagonal line of thickness 1 pixel, compare the results of morphological dilation using a structuring element defined by each distance metric's unit ball. [L4; CLO4] | 03 |
| 6. | (a) | Analyze the second-order derivatives for image sharpening with examples. [L4; CLO4] | 04 |
| | (b) | A medical X-ray image exhibits poor contrast in the region of interest (ROI) while maintaining satisfactory contrast elsewhere. Compare the effectiveness of global histogram equalization and adaptive histogram equalization for enhancing the ROI. [L4; CLO4] | 03 |
| | (c) | Write the mathematical formulas for the band-pass filter and the band-reject filter kernels, and explain how each filter affects the quality and features of an image. [L1; CLO4] | 03 |
| 7. | (a) | A two-dimensional discrete image is represented in the spatial domain by $f(x, y)$. Derive the mathematical expressions for the 2-D Discrete Fourier Transform (DFT) and the Inverse Discrete Fourier Transform (IDFT) used in frequency-domain image processing. [L3; CLO4] | 02 |
| | (b) | Discuss the fundamental properties of the 2-D DFT, and explain their significance in digital image analysis. [L5; CLO5] | 05 |
| | (c) | Compare and explain the probability density functions of Gaussian noise, Salt-and-Pepper noise, and Erlang noise commonly encountered in digital image processing. [L4; CLO5] | 03 |
| 8. | (a) | Construct the red, green, blue (RGB), and hue, saturation, intensity (HIS) color models for color image processing. [L3; CLO5] | 04 |
| | (b) | Discuss the color edge detection from the RGB image. [L6; CLO5] | 03 |
| | (c) | Explain the basic steps of the image segmentation process. [L5; CLO5] | 03 |

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5105**
 Course Title: **Fluid Dynamics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the right margin indicate full marks. The questions are of equal value.
- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Derive the Navier-Stokes equation and continuity equation for compressible viscous fluid in vector form: $\frac{\partial q}{\partial t} + q(q \cdot \nabla) = F - \frac{1}{\rho} \nabla P + \nu \nabla^2 q + \frac{1}{3} \nu \nabla(\nabla \cdot q)$. [L5; CLO1] **05**
 (b) Define Couette flow. Show that for Couette flow, the solution of Navier Stokes equation is $u = \frac{y}{h} U - \frac{h^2}{2\mu} \frac{dp}{dx} \frac{y}{h} \left(1 - \frac{y}{h}\right)$, where the symbols have their usual meaning. Also find the shearing stresses. [L2; CLO2] **05**
2. (a) What is a parallel flow? Show that for parallel flow through a straight channel, resulting velocity profile is parabolic. [L2; CLO3] **05**
 (b) Find the velocity distribution of laminar steady flow between two co-axial circular cylinders. [L1; CLO2] **05**
3. (a) Define unsteady flow. Obtain velocity distribution as well as drag coefficients for a flow past a suddenly accelerated flat plate. [L2; CLO4] **05**
 (b) An incompressible viscous fluid flows steadily parallel to the axis in an annular space between two coaxial cylinders of radii a and b ($b > a$). Show that the rate of discharge is $-\frac{\pi P}{8\mu} \left[(b^4 - a^4) - \frac{(b^2 - a^2)^2}{\log \frac{b}{a}} \right]$ **05**
 where, P is the pressure gradient. Also, find the average velocity. [L2; CLO1]
4. Show that the stream function ψ for the slow motion of viscous fluid streaming past a sphere of radius ' a ' is given by $\psi = \frac{U}{2} \left(r^2 - \frac{3}{2} ar + \frac{1}{2} \frac{a^3}{r} \right) \sin^2 \theta$. [L2; CLO3] **10**

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. Show that with a suitable substitution, the boundary layer equation over a flat plate reduces to the Blasius equation in the form: $f(\eta)f''(\eta) + 2f'''(\eta) = 0$ with boundary conditions $f(0) = f'(0) = 0$ and $f'(\infty) = 1$, 10

$$\text{where } \eta = \sqrt{\frac{U_\infty}{\nu x}} y, \psi = \sqrt{\nu U_\infty x} f(\eta)$$

Also, show that the skin friction for a plate wetted on both sides is $4f''(0)b\sqrt{\mu\rho lU^3}$, where b is the width and l is the length of the plate. [L2; CLO4]

6. Obtain the integral equation for an unsteady boundary layer flow which is related to the displacement thickness and momentum thickness. [L4; CLO5] 10
7. Find the Boundary Layer Equation for steady 2D incompressible fluid flow in a convergent/divergent channel, and hence show that 10

$$f' = \frac{U}{U_\infty} = 3 \tanh^2 \left\{ \frac{\eta}{\sqrt{2}} + \beta \right\} - 2. \quad \text{[L2; CLO6]}$$

8. Explain forced convection and free convection. Describe the parallel forced flow past a flat plate at zero incidence and hence obtain the solution for the 'cooling problem' with a prescribed temperature difference between the wall and external stream. [L6; CLO7] 10

***** THE END *****

*Levels of Bloom's cognitive domain:

[L1=Remember; L2=Understand; L3=Apply; L4=Analyze; L5=Evaluate; L6=Create]

Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0541 12 Math 5111**
 Course Title: **Optimal Control and Applications**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the margin indicate full marks. The questions are of equal value.
- Use separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

	Marks
1. (a) Briefly discuss the optimization problems and optimal control problems with examples. Distinguish the basic difference between the two types of problems. [L1; CLO1]	04
(b) Define process, optimal process, feasible solution and optimal solution of optimal control problem. Show graphically the main idea of optimal solution. [L1; CLO1]	03
(c) Describe the inter-relations among different types of optimal control problems. [L2; CLO2]	03
2. (a) Distinguish between function and functional with examples. Discuss briefly the calculus of variations technique. Derive the necessary condition for a functional of 3 rd order derivatives to be extremal. [L5; CLO3]	06
(b) State the fundamental lemma of calculus of variations. Prove the lemma by applying the 'way of contradiction (WOC)' technique. [L4; CLO2, CLO3]	04
3. (a) Define piecewise differentiable function, Lipschitz continuous function, bounded function and convex function for two variables. [L5; CLO3]	03
(b) Distinguish between algebra and σ -algebra, measure and measurable set. [L3; , CLO3, CLO4]	03
(c) State the integral form of Gronwall's lemma and prove the lemma. [L2; CLO2]	04
4. (a) Derive the necessary conditions to solve the following optimal control problem: $\min_u \int_{t_0}^{t_1} f(t, x(t), u(t)) dt$ subject to $x'(t) = g(t, x(t), u(t))$, $x(t_0) = x_0$ and $x(t_1)$ free. [L3; CLO3]	06
(b) An optimal control problem is as follows: $J(u) = \int_{t_0}^{t_1} f(t, x(t), u(t)) dt$ subject to $x'(t) = g(t, x(t), u(t))$, $x(t_0) = x_0$. Consider, $f(t, x, u)$ and $g(t, x, u)$ are both continuously differentiable functions in their three arguments and concave in x and u . Suppose u^* is a control, with associated state x^* , and λ a piecewise differentiable function, such that u^* , x^* , and λ together satisfy on $t_0 \leq t \leq t_1$: $f_u + \lambda g_u = 0$, $\lambda' = -(f_x + \lambda g_x)$, $\lambda(t_1) = 0$, $\lambda(t) \geq 0$ Then for all controls u , we have to prove that $J(u^*) \geq J(u)$. [L3; CLO3]	04

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) Discuss the main differences between necessary conditions of optimality and Pontryagin's maximum principle. Derive the set of necessary conditions for a basic optimal control problem. [L2; CLO2] 06
 (b) Prove that the Hamiltonian is a Lipschitz continuous function of time t on the optimal path. [L4; CLO1] 04

6. (a) Characterize the following optimal control problem for an optimal solution: 04

$$\min_u \int_0^2 (x(t) - t^2)^2 dt$$
 subject to $x'(t) = u(t), x(0) = 1, 0 \leq u(t) \leq 4$. [L3; CLO4]
 (b) Determine the optimal solution of the following optimal control problem: 06

$$\min_{u_1, u_2} \int_0^1 (x(t) - \frac{1}{8}u_1^2(t) - \frac{1}{2}u_2^2(t))dt$$
 subject to $x'(t) = u_1(t) + u_2(t), x(0) = 0, 1 \leq u(t) \leq 2$. [L5; CLO3]

7. (a) Consider an optimal control problem dealing with a one-sided control constraint 05

$$\max x(4) - \int_0^4 u^2(t) dt \quad \text{subject to} \quad \dot{x}(t) = x(t) + u(t), x(0) = 0, u(t) \leq 5.$$
 Write down the Hamiltonian and solve for the optimal solution. [L5; CLO4, CLO5]
 (b) Let $x(t)$ represents the number of cancer cells at time t (with exponential growth factor Q), and $u(t)$ the drug concentration. Suppose you want to minimize the number of cancer cells at the end of the treatment period and at the same time the harmful effects of the drug on the body. Formulate the optimal control problem and solve it. [L4; CLO3] 05

8. Consider a micro-parasitic infectious disease in human population, where permanent immunity to the disease can be achieved through natural recovery. Suppose you wish to minimize the number of infectious persons by a vaccination program for a certain fixed time period and at the same time to minimize the overall cost of vaccination. Formulate a FOUR compartmental SEIRN model for the vaccination schedule of this epidemic disease and solve for the optimal control u^* in closed form. [L5, L6; CLO4, CLO5] 10

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I** Examination-2026; Session: **2025-2026**
 Course Code: **0111 12 IER 5159**
 Course Title: **Research Methodology and Ethical Issues**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



- The figures in the margin indicate full marks. The questions are of equal value.
- Use separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

		Marks
1.	(a) Explain the difference between basic research and applied research. Provide one example of each from mathematical field. [L2; CLO2, L3; CLO4]	04
	(b) Discuss the role of deductive and inductive reasoning in research. Which one is more dominant in mathematics and why? [L3, L4; CLO4]	02
	(c) What is a research gap? And why is it important to know in any research? [L2; CLO1]	02
	(d) How to create a research problem based on the literature review? [L1; CLO3]	02
2.	(a) Compare quantitative and qualitative research approaches. Which is more relevant to mathematics and in what contexts? [L4; CLO2, CLO6]	04
	(b) How do you differentiate between theoretical and empirical research? [L2; CLO3]	02
	(c) Describe the key steps of writing a research proposal. [L6; CLO3]	04
3.	(a) Differentiate between a hypothesis and a conjecture. Why are conjectures more common in mathematics? [L2; CLO7]	04
	(b) Miss X is conducting research on "A Mathematical Model for the Spread of Infectious Diseases with Vaccination". Can you write objectives and conjectures? [L6; CLO10]	06
4.	(a) What is scale and measurement in Research Methodology? [L1; CLO4]	02
	(b) Differentiate between Citation, Reference and Bibliography with example. [L5; CLO9]	03
	(c) Differentiate between 'proof by contradiction' and 'proof by contrapositive'. [L5; CLO9]	05

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5.	(a) What is 'Exposition' in mathematical writing? [L2; CLO4]	02
	(b) Could you list different types of expository writing and explain any two of them. [L2; CLO4]	03
	(c) Compare and contrast between exposition and research paper. [L4; CLO4]	02
	(d) Differentiate between Citation, Reference and Bibliography with example. [L5; CLO9]	03

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|----|-----|--|----|
| 6. | (a) | What is plagiarism? [L2; CLO8] | 01 |
| | (b) | Differentiate between direct plagiarism and paraphrasing plagiarism with examples. [L3; CLO8] | 04 |
| | (c) | A student copies a proof from a textbook and changes only a few words before submission. | 05 |
| | | (i) What type of plagiarism is this? | |
| | | (ii) Why is it unethical? | |
| | | (iii) How could the student avoid plagiarism? [L3; CLO8] | |
| 7. | (a) | What is meant by authorship in research? [L2; CLO8] | 02 |
| | (b) | Explain the responsibilities of an author in mathematical research. [L4; CLO8] | 02 |
| | (c) | A professor's name is added to a paper even though they made no contribution to the research. | 06 |
| | | (i) What type of unethical authorship practice is this? | |
| | | (ii) Why is it problematic? | |
| | | (iii) What should be done instead? [L5; CLO8] | |
| 8. | (a) | What is the dissemination of research? Why is the dissemination of research important? [L5; CLO8] | 05 |
| | (b) | What are the indicators of disseminating research? How can a graphical abstract help disseminate your research? [L5; CLO8] | 05 |

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: **Mathematics**, Program: **M.Sc. in Applied Mathematics**
 Year: **MSc First**, Term: **I Examination-2026**; Session: **2025-2026**
 Course Code: **0541 12 Math 5107**
 Course Title: **Industrial Mathematics**
 Total Marks: **60**, Time: **03** Hours, Credit: **03**



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- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Define the probability density function, probability mass function and expected value and how they measure? [L1; CLO1] **04**
 (b) Define Poisson distribution. What is the relation between Poisson and Binomial distributions? [L2; CLO1] **02**
 (c) 20% chance of suffering of occupational diseases in an industry. What is the probability that out of 6 workers i) 3 ii) 4 iii) 4 or more workers will contract the diseases? [L1; CLO1] **04**
2. (a) Define Z- transformation of a sequence. Find the Z- transformation of **05**
 i. discrete unit set $U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$
 ii. $U(x) = \{15, 10, 14, 7, 4, 1, -1, 0, 3\}$ [L2; CLO2]
 (b) Define casual signals. Solve the recursion: $6u_k = 5u_{k-1} - u_{k-2}$ when $u_0 = 1$ and $u_1 = -1$. [L5; CLO2] **05**
3. (a) Define Discrete Fourier Transform (DFT) and IDFT. [L1; CLO2] **02**
 (b) If $F[k] = \{2, 1 - i, 0, 1 + i\}$ is the frequency domain signal, find its time domain signal $f[n]$. [L2; CLO2] **03**
 (c) Using convolution theorem, determine circular convolution of $f[n] = \{1, 1, 0, 0\}$ and $g[n] = -1^n, n = 0, 1, 2, 3$. [L5; CLO3] **05**
4. (a) Define marginal cost and marginal revenue. [L2; CLO3] **02**
 (b) Show that the transformation of convolution is co-ordinate wise product of the two transformations. [L3; CLO3] **03**
 (c) For what condition $\psi(x, a) = a_1\phi_1 + a_2\phi_2 + \dots + a_n\phi_n$ will be the best fit for the data. [L5; CLO4] **05**

x	x_1	x_2	$\dots$	x_n
y	y_1	y_2	$\dots$	y_n

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) What is depreciation and annuity due? Let an investment machine cost is 5,00,000Tk and its effective life is estimated 15 years. If the scrap realizes 5,000Tk only. What amount should be retained out of profits at the end of each year to accumulate at 5% compound interest per annum? [L5; CLO4] **05**
- (b) A company's production cost is $C(t)$ over time in year is governed by the DE, $\frac{dC}{dt} = 15,000 - 0.05C$, with an initial cost $C(0) = \text{Tk.}50,000$. The company's revenue is given by $R(t) = 300t + 2500$. Determine the cost function and the profit at $t = 4$. [L5; CLO4] **05**
6. (a) An attack submarine is cruising at 40 knots/hour at a depth of 1000 feet when suddenly the reactor scrams. After 1 minute way has dropped to 30 knots/hour. Construct a mathematical model of the submarine. How long does the crew have to make repairs before forward motion falls below steerageway of 2 knots/hour? [L3; CLO5] **05**
- (b) List the Big Six Partial Differential Equations (PDEs) and describe their significance in mathematical modeling. [L2; CLO5] **05**
7. (a) Explain the plant cascade model and illustrate its application in control systems. [L2; CLO5] **05**
- (b) Why numerical methods are important for solving PDEs and what types of problem are solved by this method? Using numerical method solve: $u_t = u_{xx}$, $0 < x < 1$, subject to $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 1$. [L5; CLO6] **05**
8. (a) Two cascade plants, 1st one is lowpass filter given by $\dot{y} + y = u$ and 2nd one is high-pass filter given by $\dot{z} + z = y$. Solve the system for $y(t)$ using frequency domain method. [L5; CLO6] **05**
- (b) What is Galerkin's requirement? By using Galerkin's method solve the eigen value problem: $y'' = \lambda y$, $0 < x < 1$ subject to $y(0) = 0 = y(1)$. [L5; CLO6] **05**

***** THE END *****

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Khulna University
 Science, Engineering and Technology School
 Discipline: Mathematics, Program: M.Sc. in Applied Mathematics
 Year: MSc First, Term: I Examination-2026; Session: 2025-2026
 Course Code: 0541 12 Math 5109
 Course Title: Operations Research
 Total Marks: 60, Time: 03 Hours, Credit: 03



- The figures in the margin indicate full marks. The questions are of equal value.
- Use a separate answer script for each section.

Section-A

There are **FOUR** questions in this section. Answer any **THREE** questions

Marks

1. (a) Briefly discuss about the different phases of operations research (OR). Also describe its limitations. [L1; CLO1] **05**
 (b) Solve the following linear goal programming problem: **05**
 Minimize $Z = \{o_3 + o_4, o_1, u_2, u_3 + 8/5u_4\}$,
 subject to $g_1: x_1 + x_2 + u_1 - o_1 = 20$
 $g_2: x_1 + x_2 + u_2 - o_2 = 50$
 $g_3: x_1 + u_3 - o_3 = 15$
 $g_4: x_2 + u_4 - o_4 = 8$
 $x_i, u_i, o_i \geq 0$
 Goals are written in order of priority. [L2; CLO1]
2. (a) Explain the term 'Goal Deviation' in goal programming. [L2; CLO2] **03**
 (b) Solve the following goal programming problem: A company manufactures two products, radios and televisions, which must be processed through assembly and finishing departments. Assembly has 90 hours available finishing can handle up to 72 hours of work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each television requires 3 hours in assembly and 6 hours in finishing. If profit is Tk. 120 per radio and Tk. 90 per television, determine the best combination of radios and televisions to realize a maximum profit of Tk. 2100. [L5; CLO2] **07**
3. Solve the following mixed integer programming problem: **10**
 Maximize $z = 6x_1 + 9x_2$
 Subject to $-x_1 + 3x_2 \leq 6$
 $7x_1 + x_2 \leq 35$
 $x_1, x_2 \geq 0$ and x_1 an integer. [L1; CLO4]
4. (a) Briefly explain about dynamic programming problem. [L4; CLO4] **03**
 (b) Evaluate optimal solution of the following problem with help of dynamic programming process: **07**
 $Min Z = y_1^2 + y_2^2 + y_3^2$
 Subject to: $y_1 + y_2 + y_3 = 10; y_1, y_2, y_3 \geq 0$. [L6; CLO4]

Section-B

There are **FOUR** questions in this section. Answer any **THREE** questions

5. (a) What do you mean by decision theory? Discuss taking decision making under certainty, under risk, and under uncertainty. [L1; CLO1] 04
- (b) The following matrix gives the payoff of different alternatives a_1, a_2 and a_3 against states of nature (events) S_1, S_2, S_3 and S_4 . Discuss the decision taken under the following approach (i) Pessimistic (ii) Optimistic [L4; CLO2] 06

	S_1	S_2	S_3	S_4
a_1	Tk. 4,000	Tk. -100	Tk. 6,000	Tk. 18,000
a_2	Tk. 20,000	Tk. 5,000	Tk. 400	Tk. 0
a_3	Tk. 20,000	Tk. 15,000	Tk. -2,000	Tk. 1,000

6. (a) Solve the following 2×2 game by arithmetic method: [L3; CLO4] 05

		Player B	
Player A		B_1	B_2
	A_1	2	-1
	A_2	-1	0

- (b) Solve the following game by graphical method: [L5; CLO5] 05

		Player B			
Player A		B_1	B_2	B_3	B_4
	A_1	2	1	0	-2
	A_2	1	0	3	2

7. (a) Use dominance rule to reduce the following game to 2×2 game and hence find the optimum strategies and the value of the game for player A and B. [L3; CLO6] 05

	B_1	B_2	B_3
A_1	3	-2	4
A_2	-1	4	2
A_3	2	2	6

- (b) Two players A and B play the following game. A has a bag containing three coins of 1 Tk, 2 Tk, and 5 Tk. A Takes one coin from the bag and before it is exposed, B guesses what it is? If he, B is right he takes the coin if he is wrong, he gives to A a coin of the same taka. Solve the game. [L4; CLO6] 05

8. Solve the following non-linear programming problem (NLPP) by using Kuhn-Tucker conditions 10

$$\text{Optimize } Z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$$

$$\text{Subject to: } x_1 + x_2 \leq 1; 2x_1 + 3x_2 \leq 6; x_1, x_2 \geq 0. \text{ [L6; CLO5]}$$

***** THE END *****

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